For particle motion in one dimension

If force is function of position x only
$F = F(x)$

Work done by force as particle goes $x_1 \rightarrow x_2$:
- Independent of how it goes (i.e., history, path, etc.)
- Depends only on end points x_1, x_2

$V(x)$ define as:

$$V(x) = -\int_{x_R}^{x} F(x)dx$$

- Is function of x only
- Independent of how it goes $x_R \rightarrow x$
- Scalar field: Potential Energy

May write:

$$F(x) = -\frac{dV(x)}{dx}$$

Quantity $E = T + V(x)$ conserved
E = Total Mechanical Energy

Since E is conserved
Force is called *conservative*

Non-conservative forces

- Total Mechanical Energy is NOT conserved
- Work depends on the how the particle goes $x_1 \rightarrow x_2$ (i.e., Work depends on history, path, etc.)
- NOT possible to define scalar field Potential Energy function that is path independent and that depends only on position.

Solution of Newton's equation in one dimension for force that depends only on position: $F=F(x)$

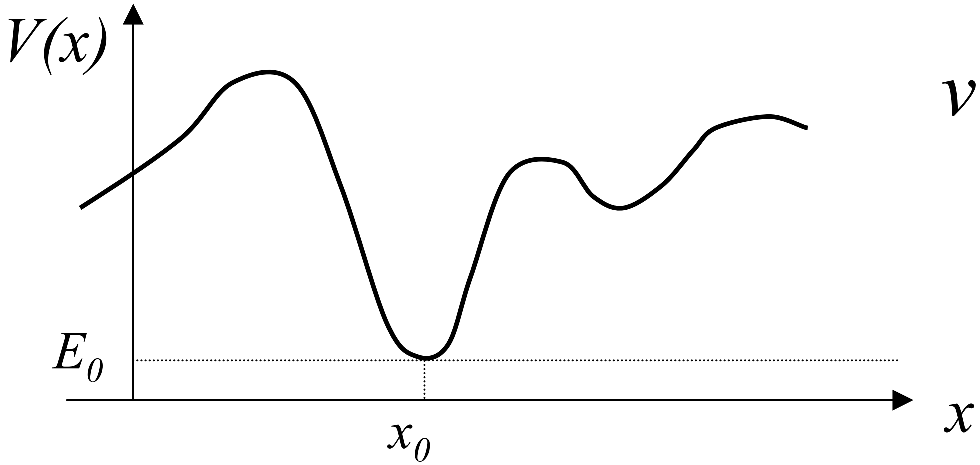
Defined Potential Energy Function: $$V(x) = -\int_{x_R}^{x} F(x)\,dx$$

$$T + V(x) = E \Rightarrow$$

Energy integral: $$t - t_0 = \pm \int_{x_0}^{x} \frac{dx}{\sqrt{\frac{2}{m}\left[E - V(x)\right]}}$$

Leads to solution: $x(t)$

Learning about *x(t)* from energy integral



$$v = \pm\sqrt{\tfrac{2}{m}\left[E - V(x)\right]} \Rightarrow$$

Motion allowed only for

$$E \geq V(x)$$

If initial conditions (x_0, v_0) are such that $E = E_0$

<u>What to expect?</u>

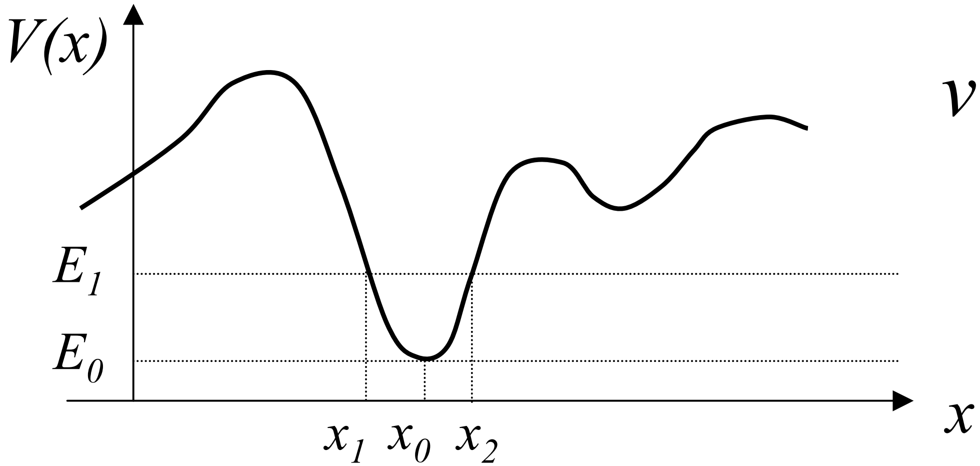
$$E_0 \geq V(x) \Leftrightarrow x = x_0$$

$$[E_0 = V(x) \Rightarrow v = 0]$$

Particle can only be at rest at x_0 !

Force at $x = x_0$: $F(x_0) = -\left(\dfrac{dV(x)}{dx}\right)_{x=x_0} = 0$ **No acceleration !**

Learning about *x(t)* from energy integral



$$v = \pm\sqrt{\tfrac{2}{m}\left[E - V(x)\right]} \Rightarrow$$

Motion allowed only for

$$E \geq V(x)$$

If initial conditions (x_0, v_0) are such that $E = E_1 > E_0$

<u>What to expect?</u>

$$E_1 \geq V(x) \Rightarrow x_1 \leq x \leq x_2$$

$$\begin{cases} E_1 > V(x) \Rightarrow v > 0 \ or \ v > 0 \\ E_1 = V(x) \Rightarrow v(x_1) = v(x_2) = 0 \end{cases}$$

-Particle can move between x_1 & x_2

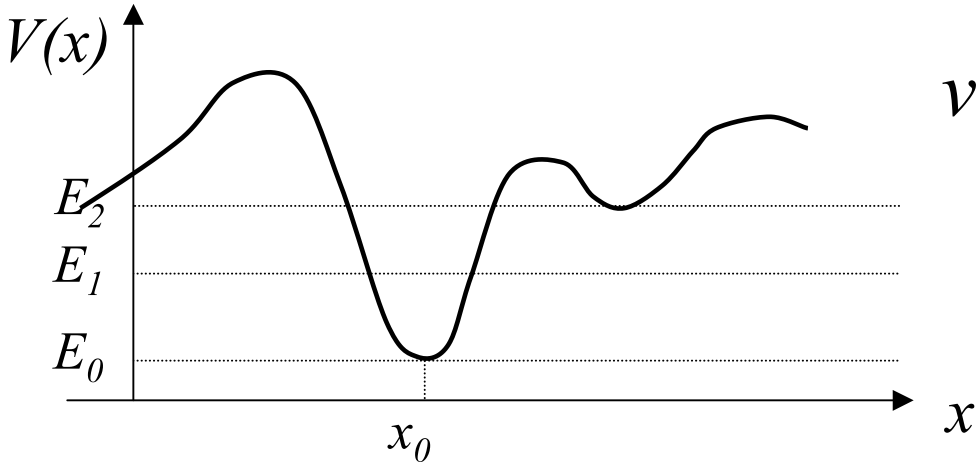
- Stops at x_1 & x_2: Turning points

Force at x_1, x_0, x_2:

$$F(x) = -\left(\frac{dV(x)}{dx}\right)$$

- $F \neq 0$ at x_1 & x_2
- $F = 0$ at x_0

Learning about *x(t)* from energy integral



$$v = \pm\sqrt{\tfrac{2}{m}\left[E - V(x)\right]} \Rightarrow$$

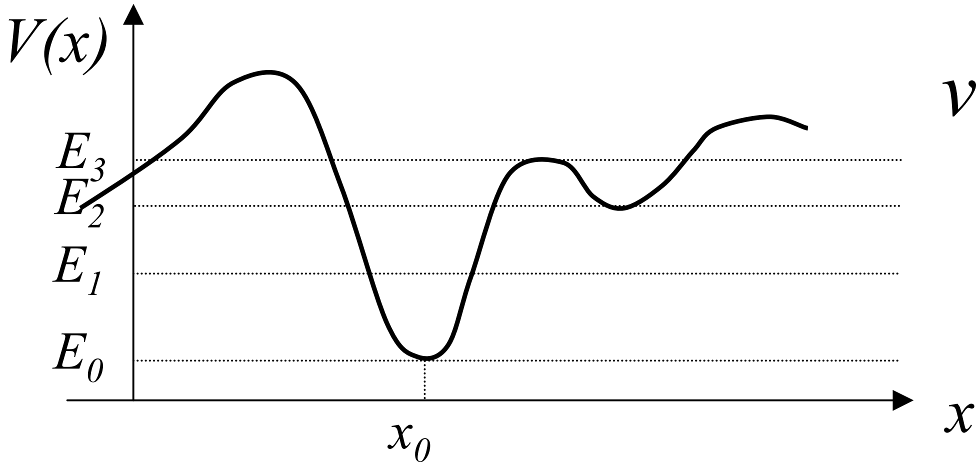
Motion allowed only for

$$E \geq V(x)$$

If initial conditions (x_0, v_0) are such that $E = E_2 > E_1$

<u>What to expect?</u>

Learning about *x(t)* from energy integral



$$v = \pm\sqrt{\tfrac{2}{m}\left[E - V(x)\right]} \Rightarrow$$

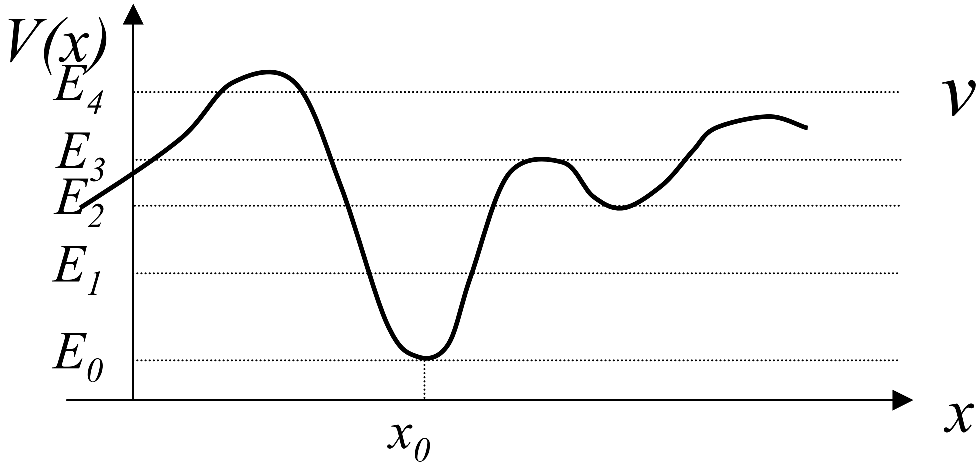
Motion allowed only for

$$E \geq V(x)$$

If initial conditions (x_0, v_0) are such that $E = E_3$

<u>What to expect?</u>

Learning about *x(t)* from energy integral



$$v = \pm\sqrt{\frac{2}{m}\left[E - V(x)\right]} \Rightarrow$$

Motion allowed only for

$$E \geq V(x)$$

If initial conditions (x_0, v_0) are such that $E = E_4$

<u>What to expect?</u>

Example of working a problem with *F=F(x)*

A particle of mass m moves in one dimension under the force:

$$F(x) = -\frac{GMm}{x^2} \qquad G,\ M,\ m \text{ are positive constants}$$

a) Discuss the effect of this force on the total mechanical energy of the particle. Is it appropriate to define a potential energy for the motion of this particle? Why?

b) Find an expression for the potential energy $V(x)$ of the particle (Choose a reference point such that any arbitrary constants vanish)

c) Draw by hand a sketch of the potential energy $V(x)$
(No computer use upfront, please!)
(You may check with a computer afterwards)

d) For which values of the total energy will the motion be:
-Bound (i.e., confined). Find the turning points.
-Unbound, with change of direction. Find the turning points.
-Unbound, with no change of direction.